$$\therefore \frac{dV}{dx}=s-3\pi x^2=0. \ \therefore \ x=\sqrt{\left(\frac{s}{3\pi}\right)}, =y.$$

Also solveu by *P. S. BERG, C. W. M. BLACK, J. H. DRUMMOND, A. HUME, P. H. PHILBRICK, H. C. WHITAKER, G. B. M. ZERR*, and the *PROPOSER.*

PROBLEMS.

20. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

$$\int_0^{\frac{1}{2}\pi} \surd[(1-e^2\cos^2\phi)(1-e^2\sin^2\phi)]d\phi=\text{what?}$$

21 Proposed by T. JOHN COLE, Columbus, Ohio.

In the equilateral triangle ABC, AB the base is 10 feet. With B as a center an arc is drawn from C to A; likewise with A as a center an arc is drawn from C to B. What is the volume of the solid generated by revolving the figure about the altitude of the triangle as an axis.

Solutions to these problems should be received on or before August 1st.

MECHANICS.

Conducted by B.F.FINKEL,Kidder, Missouri. All Contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

6. Proposed by THOMAS W. WRIGHT, M. A., Ph. D., Professor of Applied Mathematics and Physics, Union College, Schenectady, New York.

What is the effect of a charge between light and heavy cavalry, the light cavalry having the greater energy and the heavy the greater momentum?

Solution by P. H. PHILBRICK. C. E., Lake Charles, Louisiana.

Let M and V represent the mass and velocety repectively of the heavy cavalry and M_1 and V_1 the same of the light cavalry. Supposing the bodies to be inelastic and moving in opposite directions before impact and together after impact their common velocity after impact is, $V=\dfrac{MV-M_1V_1}{M+M_1}$ ....(1).

Since $MV>M_1V_1$ the heavy cavalry will overcome the lighter and carry it along at the above rate. The combined energy of the bodies is, $\frac{1}{2}MV^2+\frac{1}{2}M_1V_1^2$....(2). This measures the distructive effect of the charge.

7. Proposed by DE VOLSON WOOD, M. A., M. Sc., C. E., Professor of Engineering, Stevens Institute of Technology, Hoboken, New Jersey.

A hollow sphere filled with frictionless water rolls down a rough plane whose length is l and inclination θ; when half way down the water suddenly freezes and adheres to the sphere. Required the time of the decent.

I. Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics in the University of Mississippi.

Let M=the mass of the hollow sphere;
m=the mass of the water;
R=the radius of the outer surface of the sphere;
r=the radius of the inner surface of the sphere;
K=the radius of gyration of the shell about a diameter;
k=the radius of gyration of the water about a diameter;
l=the length of the inclined plane;
α=the inclination of inclined plane;
x=the distance from the upper end of the plane to the point where the sphere touches it after t seconds;
θ=the angle turned through;
F=the friction acting upon the plane.

The equations of motion are

$$(M+m)\frac{d^2x}{dt^2}=(M+m)g.\sin\alpha-F\ldots.(1).$$

and, since the water, being frictionless, does not partake of the rotary motion of the shell,

$$MK^2\frac{d^2\theta}{dt^2}=FR\ldots.(2).$$

Also, the plane being perfectly rough, there is no sliding. Hence,

$$x=R\theta,\text{ and }\frac{d^2x}{dt^2}=R\frac{d^2\theta}{dt^2}\ldots.(3).$$

From equations (1), (2), and (3)

$$\frac{d^2x}{dt^2}=\frac{(M+m)R^2g\sin\alpha}{(M+m)R^2+MK^2}.$$

This being the acceleration, the space passed over from rest in t seconds is given by

$$x=\tfrac{1}{2}\frac{(M+m)R^2g\sin\alpha}{(M+m)R^2+MK^2}t^2.$$

The time required to traverse the upper half of the plane is, therefore,

$$\sqrt{\frac{(M+m)R^2+MK^2}{(M+m)R^2g\sin\alpha}l}.$$

and the velocity acquired, v, is

$$\sqrt{\frac{M(+m)R^2g\sin\alpha}{(M+m)R^2+MK^2}l}.$$

Now let

w=angular velocity immediately before the water freezes;
w'=angular velocity immediately after the water freezes;
v'=velocity of center immediately after the water freezes;

Since any change in the motion is due to an impulse at the point of tangency of the sphere with the plane, the angular momentum about a hori-

zontal line in the plane and through this point is not altered. Therefore

$$(M+m)v.R+MK^2.w=(M+m)v'.R+(MK^2+mk^2)w'.$$

Then, since $v=Rw$ and $v'=Rw'$,

$$v=\frac{(M+m)R^2+MK^2}{(M+m)R^2+MK^2+mk^2}v$$

$$=\frac{\sqrt{[(M+m)R^2+MK^2](M+m)R^2\,g\sin\alpha.l}}{(M+m)R^2+MK^2+mk^2}$$

For the motion down the lower half of the plane the equations are,

$$(M+m)\frac{d^2x}{dt^2}=(M+m)g\sin\alpha-F'$$

and $(MK^2+mk^2)\dfrac{d^2\theta}{dt^2}=F'R$ where F' is the friction.

$$\text{Also, } x=R\theta,\ \frac{d^2x}{dt^2}=R\frac{d^2\theta}{dt^2}.$$

Solving these for t, remembering that when $t=0$, $\dfrac{dx}{dt}=v'$, the time is found to be

$$-\sqrt{\frac{(M+m)R^2+MK^2}{(M+m)R^2g\sin\alpha}l}+\sqrt{\frac{(M+m)R^2+MK^2+(M+m)R^2+MK^2+mk^2}{(M+m)R^2g\sin\alpha}l}.$$

The whole time, therefore, is

$$\sqrt{\frac{2(M+m)R^2+2MK^2+mk^2}{(M+m)R^2g\sin\alpha}l}.$$

Since $K^2=\dfrac{2}{5}.\dfrac{R^5-r^5}{R^3-r^3}$ and $k^2=\dfrac{2}{5}r^2$, this becomes

$$\sqrt{\frac{10(M+m)(R^3-r^3)R^2+4M(R^5-r^5)+2m(R^3-r^3)r^2}{5(M+m)(R^3-r^3)R^2g\sin\alpha}l}.$$

Also solved by *P. H. PHILBRICK*, *WILLIAM HOOVER*, and *J. C. NAGLE*. We will publish one or all of these excellent solutions in the next issue.

PROBLEMS.

13. Proposed by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

A man, horse and buggy are going around a circular race course at a 2:40 gait. If the whole outfit weighs 1500 lbs, radius of course is 500 feet and track is inclined so that pressure is equal upon the wheels, find the pressure on the ground due to whole weight.

14. Proposed by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University P. O.

"The center of a sphere of radius c moves in a circle of radius a and generates thereby a solid ring, as an anchor ring; prove that the moment of inertia of this ring about an axis passing through the center of the direct circle and perpendicular to its plane is $\dfrac{\pi^2\rho ac^2}{4}(4a^2+3c^2)$."